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About the Dynamics of a Dynamic System

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ABSTRACT

This article provides brief information about dynamic systems. We find analytical and numerical solutions, and fixed points of the continuous-time generalized quadratic stochastic Volterra operator (dynamical system). We use the theory of ordinary differential equations for an analytical solution to the dynamic system. The eigenvalues of the parameters indicate the origin of the solution of the previously analyzed dynamical system. Our results extend and improve the corresponding recent results in the literature numerically and analytically. Moreover, we use Runge-Kutta method for finding our results.

Keywords: Quadratic Stochastic Operators; Dynamic System; Numerical Solution; Analytical Solution; Fixed Point.

1. Introduction

The development of living beings manifests itself differently in different processes. This is influenced by fertility, growth, personality, death of individuals, external environment and the like. Taking these circumstances into account, a mathematical model of the population will be constructed.

Changes in the number of the population make up its dynamics. Population dynamics is part of mathematical biology and is a “mathematical polygon” aimed at determining the state of a population over time. Because mathematical modeling makes it possible to obtain complete information about the process being studied and to draw a conclusion about its growth or decline.

In general, mathematical models of processes consist of systems of simple differential equations and inequalities, with the help of their solutions; it is possible to predict the change in the influence of one or another factor. For example, if the population process is represented by systems of ordinary differential equations, the necessary results can be obtained by qualitative analysis of such equations. In particular, without finding a solution to a system of equations, it is possible to have accurate information about its trajectory and similar properties [1].

Quantitative dynamics of the population are studied in mathematical models in connection with its gender and age structure, the influence of the external environment, the genetic form under the influence of various factors of evolution, and the results of human activity. There are many dynamic processes in the inanimate world. They are also easy to model. But it is relatively difficult to create dynamic models for living organisms. Therefore, static models are used before creating dynamic models, that is, a number of factors are idealized, some are not taken into account. Examples of static models are trying to explain the arrangement of plant leaves or the structure of mollusk shells using the pattern of spiral lines.

Since the population is variable, scientists are interested not only in a certain change in its number and density, but also in what factors it changes, that is, its dynamics. A dynamic description of the population is defined by births, productivity, deaths, emigration and immigration, among others. True populations have different rates of reproduction and mortality in different groups. For example, insects lay eggs and their enemies kill the larvae, in addition, they are influenced by metabolic products in the environment, cannibalism and poisoning, age stages and their intensity.

Analysis of population dynamics, in turn, is important both theoretically and practically. Methods of combating the reproduction of insect pests (releasing infertile male insects, pheromone traps, etc.) are aimed at creating a certain imbalance in the population structure, leading to both a slowdown and disruption of their reproduction. These questions are among the most pressing in epidemiology.

2. Analysis of Literature

We will present an analysis of scientific research conducted by the author and the results obtained to show the difference between the issue under study and other issues. To present the analysis of scientific research, we provide information about scientific articles carried out mainly on dynamic continuous-time systems.

In work [2], to study the control of ovulation in mammals, for example, to regulate a given number of mature eggs, a model of differential equations is used.

When studying the bifurcation properties of the predator-prey system with shelter and permanent prey, [3] uses systems of ordinary differential equations. It is established that the number of equilibrium and the properties of the system will change due to refuge and harvesting, and also the rich dynamics of the system compared to the system without shelter and harvesting are obtained.

The book [4] outlines the theories of dynamical systems, in particular the theory of dynamical systems with continuous time. For example, mathematical models of predator-prey of Lotka-Volterra and Gause, the spread of Kermack-Makkendrick epidemics, models of the evolution of gene families, etc.

In [5], the delayed Holling-Tanner predator-prey system with a functional response was analyzed. The existence of the Hopf bifurcation is investigated. By deriving the equation describing the flow on the central manifold, the direction of the Hopf bifurcation and its stability are studied. Numerical solutions are presented for comparison with analytical ones.

In [6], the Holling-Tanner system is studied. Using the construction of the Lyapunov function, sufficient conditions for the global stability of positive equilibrium are found. In addition, by changing the variable, the Holling-Tanner model was transformed into the studied Lienard equation.

In article [7], a continuous analogue of strictly non-Volterian quadratic dynamic systems with continuous time and equilibrium points was studied, a phase portrait of the system was constructed, numerical solutions were found, and a comparative analysis with a particular solution of the system was carried out. In [8], fixed points and general solutions of a dynamical system with continuous time were found and their asymptotic behavior of trajectories was studied.

If we analyze quadratic stochastic operators with discrete and continuous time, we will come to the conclusion that both have their advantages and disadvantages. Thus, dynamic systems with discrete time consider the states of evolution or the trend of development of processes taking into account the initial state, ignoring the interaction between individuals. For example, a child's current performance score is related to his and his mother's score the month before. Discrete-time models treat this interaction between mother and child as if it ceases to exist between dimensions and as if they interact at several discrete moments rather than continuously.

In reality, «mothers and children» do not cease to exist between classes ...». If these scores (the child's score and the mother's score) are imagined to represent two variables (perhaps monthly interactions between mother and child), then discrete-time approaches can lead to models that are theoretically interesting [9].

In [10], the flows of dynamic systems with continuous time, having the same number of equilibrium points and trajectories and not having a periodic orbit, were studied; they form an equivalence class under the condition of a topological conjugacy relation. It is proved that arbitrary two flows of the studied dynamical systems have the same dynamics in time and other properties, such as order isomorphism and homeomorphy.

[11-12] articles also studied population dynamics for the continuous time case of the model. According to the results obtained, mathematical models built on the basis of the system of ordinary differential equations considered in the articles do not give positive (since the reproduction process is discrete) results in representing the dynamics of seasonal reproduction of many other species. Mathematical models that represent these types of processes using a system of impulse ordinary differential equations [13] are the most preferred models and represent the process under study close to the real process. In particular, in biology, the mathematical model of population evolution is represented by quadratic stochastic operators [14].

3. Main Part

The article, devoted to the study of the dynamics of a bipolar population [15], gives a definition of its mathematical model and presents representations of quadratic stochastic operators representing the population.

The general evolution equation is constructed as follows. We assume that the population under consideration is two-sex. $\mathcal{F} = \{F_1, \dots, F_n\}$ is a set of female type, $\mathfrak{M} = \{M, \dots, M_\nu\}$ is a set of male type. $n + \nu$ is the population size. The state of the population is defined as the pair of probability distributions in the sets $\mathcal{F}$ and $\mathfrak{M}$ for the pair $x = (x_1, \dots . x_n)$ and $y = (y_1, \dots, y_\nu)$:

$$x_i \geq 0, \sum_{i=1}^{n} x_i = 1;\ y_k \geq 0, \sum_{i=1}^{\nu} y_k = 1.$$

Cartesian product of $\Delta^{n-1} - (n-1)$ and $\Delta^{\nu-1} - (\nu - 1)$ dimensional simplex gives the state space of the population $S = \Delta^{n-1} \times \Delta^{\nu-1}$.

If, as a result of crossing F' and natural selection in an arbitrary state $z = (x, y)$, a single value $z' = (x', y')$ is determined in generation F, then the differentiation of this population is called hereditary.

$$z' = Vz\ (z \in S)$$

the mapping $V: S \to S$ defined by the equation is called the evolution operator, and the equation is called the population evolution equation. This reflection becomes the following system of equations in coordinates:

$$x_i' = f_i(x_1, \dots, x_n: y_1, \dots, y_\nu)\ (1 \le i \le n,$$

$$y_i' = g_i(x_1, \dots, x_n: y_1, \dots, y_\nu)\ (1 \le k \le \nu.$$

We present information about the quadratic stochastic operator representing the population (two-gender population) introduced by Uzbek scientists [16]. Quadratic stochastic operators of a free population have the following meaning: we assume that the free population consists of m elements. In that case

$$S^{m-1} = \{x = (x_1, \dots, x_m) \in R^m, x_i \ge 0, i = \overline{1,n}\}, \sum_{i=1}^{m} x_i = 1,$$

the set $(m-1)$ is called a simplex of size.

The quadratic stochastic operator, self-reflecting simplex S^{m-1}, has the form $V: S^{m-1} \to S^{m-1}$, $V: x_k{}' = \sum_{i,j=1}^{m} p_{ij,k} x_i\, x_j, k = 1, \dots, m$, where $p_{ij,k} -$ is the heritability coefficient and

$$p_{ij,k} \ge 0, \sum_{i,j=1}^{m} p_{ij,k} = 1, i, j, k = 1, \dots, m.$$

For $x^{(0)} \in S^{m-1}$ quadratic stochastic operator $\{x^{(n)}\}, n = 0,1,2,3, \dots.$ trajectory is found by the expression $x^{(n+1)} = V(x^{(n)}), n = 0,1,2,3, \dots$

We assume that the state of $G-$generation is (x, y). Its next state is determined by the following formulas:

$$W := \begin{cases} x_j{}' = \sum_{i,k=1}^{n,\nu} p_{ik,j}^{(f)}\, x_i y_k\ 1 \le j \le n, \\ y_l{}' = \sum_{i,k=1}^{n,\nu} p_{ik,l}^{(m)}\, x_i y_k, 1 \le l \le \nu. \end{cases}$$

Definition ([16]). Evolutionary operator W is called a quadratic Volterra-type stochastic operator if $p_{ik,j}^{(f)} = 0, j \notin \{i, k\}\ 1 \le i, j \le n, 1 \le k \le \nu$ and $p_{ik,l}^{(m)} = 0, l \notin \{i, k\}, 1 \le i \le n, 1 \le k, l \le \nu$.

Note that when deriving evolutionary equations, it was not assumed that differentiation was genotypic (although in applications this is the main case), but only the existence of certain coefficients of heredity and survival, which ensured the hereditary nature of differentiation.

We study a generalized continuous-time analogue of one of the discrete-time quadratic stochastic operators representing a bipolar population, introduced in [16].

Let's assume that $n = \nu = 2$. Then the continuous time analogue of the discrete quadratic stochastic operator on the edge of the eighth square representing the population will be the following:

$$\begin{cases} \dot{x}_1(t) = 0, \\ \dot{x}_2(t) = 0, \\ \dot{y}_1(t) = x_1(t)y_1(t) - y_1(t), \\ \dot{y}_2(t) = x_2(t)y_1(t). \end{cases} \quad (1)$$

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A generalized case of this system is the following

$$\begin{cases} \dot{x}_1(t) = 0, \\ \dot{x}_2(t) = 0, \\ \dot{y}_1(t) = ax_1(t)y_1(t) - y_1(t), \\ \dot{y}_2(t) = bx_2(t)y_1(t) \end{cases} \quad (2)$$

Let's carry out a qualitative analysis of the system of equations, here $a > 0, b > 0$.

From the conditions of the problem it follows that $x_1(t) \geq 0,\ x_2(t) \geq 0,\ y_1(t) \geq 0,\ y_2(t) \geq 0$ and $x_1(t) + x_2(t) = 1,\ y_1(t) + y_2(t) = 1$. If $a = b = 1$, we get system (1).

It is known that the equilibrium state of the dynamic system corresponds to the critical (singular, fixed) points of the differential equation, and the closed-phase curves correspond to its periodic solutions.

(2) system has the following general solutions:

$$1) \begin{cases} x_1 = c_1, \\ x_2 = c_2, \\ y_1 = c_3, \\ y_2 = bc_2c_3t + c_4, \end{cases} \quad 2) \begin{cases} x_1 = c_1, \\ x_2 = c_2, \\ y_1 = c_3, \\ y_2 = c_4\,, \end{cases} 3) \begin{cases} x_1 = c_1, \\ x_2 = 0, \\ y_1 = c_3e^{(ac_1-1)t}, \\ y_2 = c_4, \end{cases} \quad 4) \begin{cases} x_1 = c_1, \\ x_2 = c_2, \\ y_1 = c_3e^{(ac_1-1)t}, \\ y_2 = \dfrac{bc_2c_3}{ac_1-1}e^{(ac_1-1)t} + c_4, \end{cases} \quad (3)$$

here $c_1, c_2, c_3, c_4 -$ are non-negative fixed numbers, $c_1 + c_2 = 1,\ c_3 + c_4 = 1$. The solution of the system in case 4 is more general than the other solutions. Given that the condition of the problem is $x_1 + x_2 = 1$ and $y_1 + y_2 = 1$ (for the case $c_3 \neq 0$), we get

$$\frac{bc_2}{ac_1 - 1} = -1 \quad (4)$$

and the solution will look like this:

$$\begin{cases} x_1 = c_1, \\ x_2 = c_2, \\ y_1 = c_3e^{(ac_1-1)t}, \\ y_2 = -c_3e^{(ac_1-1)t} + c_4 \end{cases} \qquad \left(\begin{cases} x_1 = c_1, \\ x_2 = 1 - c_1, \\ y_1 = 0, \\ y_2 = 1. \end{cases} \text{when } c_3 = 0 \right). \quad (5)$$

So, the general solution looks like this:

$$\begin{cases} x_1 = c_1, \\ x_2 = 1 - c_1, \\ y_1 = c_3e^{(ac_1-1)t}, \\ y_2 = -c_3e^{(ac_1-1)t} + 1. \end{cases} \quad (6)$$

Let's study the case where $a = b$ is divided into two: $a = b = 1$ and $a = b \neq 1$. In the first case, the solution of the system will have the following form:

$$1) \begin{cases} x_1 = c_1, \\ x_2 = c_2, \\ y_1 = c_3, \\ y_2 = (1 - c_1)c_3t + c_4; \end{cases} \quad 2) \begin{cases} x_1 = c_1, \\ x_2 = c_2, \\ y_1 = 0, \\ y_2 = 1; \end{cases} \quad 3) \begin{cases} x_1 = c_1, \\ x_2 = 1 - c_1, \\ y_1 = c_3e^{(c_1-1)t}, \\ y_2 = -c_3e^{(c_1-1)t} + c_4. \end{cases} \quad (7)$$

Note 1. In (7) condition (4) is self-fulfilling.

Now we see the case $a = b \neq 1$. If we take $ax_1 = u_1, ax_2 = u_2$, the system will look like this:

$$\begin{cases} \dot{u}_1 = 0, \\ \dot{u}_2 = 0, \\ \dot{y}_1 = u_1 y_1 - y_1, \\ \dot{y}_2 = u_2 y_1. \end{cases}$$

This case has been fully studied above.

Now we analyze the solution in case 4, where $a \neq b$. It is theoretically important that $ac_1 - 1 < 0$ for the desired solution, since the solution tends to finite values. According to the condition of the problem, since $0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1,\ x_1 + x_2 = 1$ it follows that $0 \leq c_1 \leq 1, 0 \leq c_2 \leq 1,\ \ c_1 + c_2 = 1$. From (4) we find $c_1 = (1-b)/(a-b)\ \Rightarrow c_2 = (a-1)/(a-b)$. From the condition of the problem, we get the following:

$$0 \leq \frac{1-b}{a-b} \leq 1, \qquad 0 \leq \frac{a-1}{a-b} \leq 1.$$

We find that for $-b > 0\ A)\ a \geq 1, b \leq 1$; for $-b < 0\ B)\ a \leq 1, b \geq 1$. Now let's define the sign $ac_1 - 1$.

$$a\frac{1-b}{a-b} - 1 < 0 \Rightarrow \frac{1-a}{a-b} < 0.$$

In the cases $a > 1, b < 1$ and $a < 1, b > 1$ there will be $ac_1 - 1 < 0$. Here we can put equality on the one hand, since the case $a = b = 1$ is considered separately. Therefore, in $a > 1, b \leq 1$ and $a \geq 1, b < 1$, the condition $ac_1 - 1 < 0$ is satisfied.

The system has the following fixed points: $M_1(c_1, c_2, 0, c_4)$ and $M_2(1/a, 0, c_3, c_4)$.

Considering that $x_1 + x_2 = 1,\ y_1 + y_2 = 1$, we find the fixed points $M_1(c_1, 1-c_1, 0, 1)$ and $M_1(c_1, 1-c_1, 0, 1)$, when $a = b = 1$. However, without $a_1 \neq 1$ the system will have only one fixed point $M_1(c_1, 1-c_1, 0, 1)$.

(2) using the linearity of the system, we determine that in the case of $a = 1$: $M_1 -$ is unstable, $M_2 -$ is stable (but not asymptotically stable), and in the case of $a \neq 1$: $M_1(c_1, 1-c_1, 0, 1)$is a stable (but not asymptotically stable) point.

The numerical solution of system (2) for the case $a = b = 1$ was found using the MathCAD mathematical system (Fig. 1, 2, 3, 4). The initial values were obtained as follows:

$$C1 = \begin{pmatrix} 0.2 & 0.31 & 0.222 & 0.42 & 0.44 & 0.5 & 0.6 & 0.71 & 0.54 & 0.77 \\ 0.8 & 0.69 & 0.778 & 0.58 & 0.56 & 0.5 & 0.4 & 0.29 & 0.46 & 0.23 \\ 0.45 & 0.14 & 0.13 & 0.24 & 0.15 & 0.35 & 0.59 & 0.28 & 0.49 & 0.46 \\ 0.55 & 0.86 & 0.87 & 0.76 & 0.85 & 0.65 & 0.41 & 0.78 & 0.51 & 0.54 \end{pmatrix},$$

herein $C1 = (C_2, C_2; C_3, C_4)^T$.

Graphs of numerical solutions are presented in Figures 1, 2, 3, 4.

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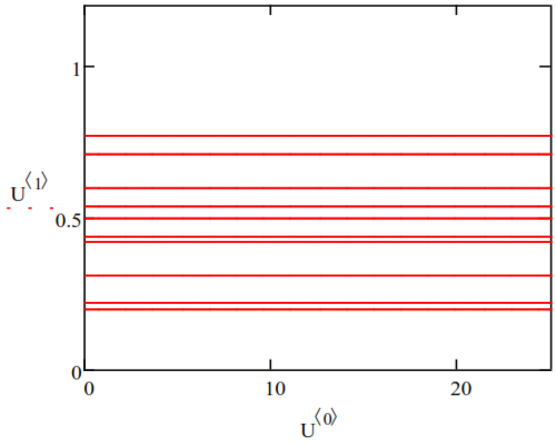

Figure 1

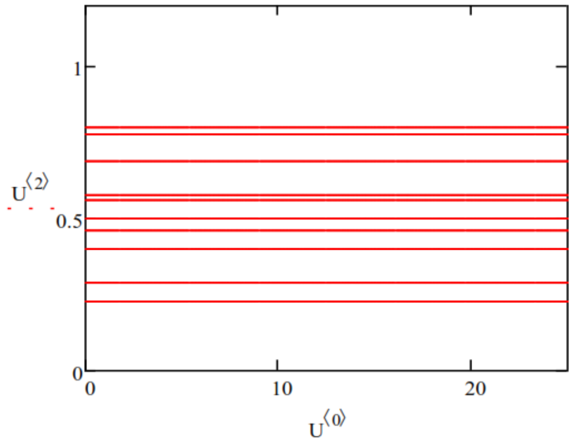

Figure 2

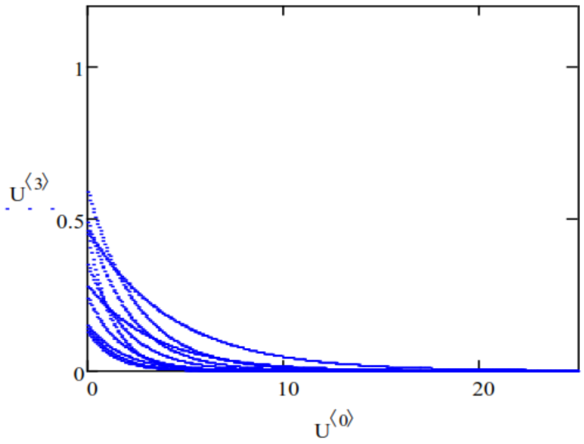

Figure 3

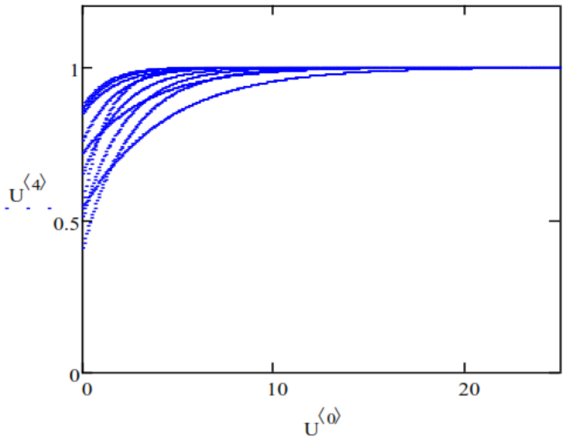

Figure 4

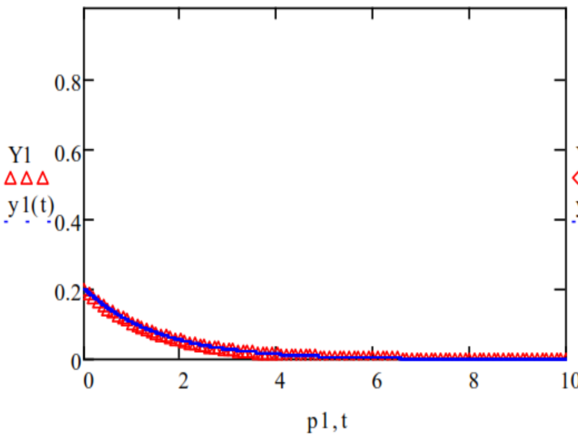

Figure 5

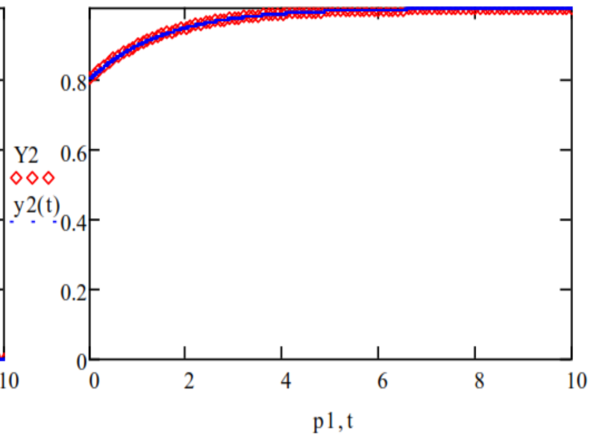

Figure 6

$U^{<0>}$ – interval t – time, $U^{<1>}$ – through $x_1(t)$, $U^{<2>}$ – through $x_2(t)$, $U^{<3>}$ – through $y_1(t)$, $U^{<4>}$ – - defined through $y_2(t)$.

In the mathematical system MathCAD (2), analytical and numerical solutions of the system were compared. As a result, it was found that the mutual difference between the analytical and numerical solutions up to $t \leq 7$does not exceed 0.001, and after $t \geq 7$ it coincides. For given initial values, it was observed that the solution tends to the fixed point $M_1(0,1;0,1)$. The graphs in Figures 5-6 are drawn at the initial values $C_1 = 0.35, C_2 = 0.65, C_3 = 0.2, C_4 = 0.8$.

Where $Y1$ $(Y2)$ is the numerical solution of the system, $y1(t)(y2(t)) -$ is the analytic solutions, $p1$ and $t -$ are the programming steps.

Now, for cases where $a \neq 1$ and $b \neq 1$, we look for (1) numerical solutions of the system. $a = 1.1,\ b = 0.98$ and

$$C2 = \begin{pmatrix} 0.32 & 0.1 & 0.22 & 0.42 & 0.19 & 0.5 & 0.26 & 0.7 & 0.5 & 0.77 \\ 0.68 & 0.9 & 0.78 & 0.58 & 0.81 & 0.5 & 0.74 & 0.3 & 0.5 & 0.23 \\ 0.47 & 0.4 & 0.3 & 0.24 & 0.36 & 0.35 & 0.159 & 0.25 & 0.33 & 0.37 \\ 0.53 & 0.6 & 0.7 & 0.76 & 0.64 & 0.65 & 0.841 & 0.75 & 0.67 & 0.63 \end{pmatrix}$$

in the case of (1) we draw graphs of numerical solutions of the system (Graphs 7-10).

Note 2. Other cases of a and b (for example, $a < b$; $a = 1, b \neq 1$ etc.) are studied in the same way as mentioned above. It does not differ much from the case presented in the article.

Note 3. Analyzing Figures 3-6, it was found that the solutions of system 1 tend to a fixed point at $t \geq 7$ at different initial values. This situation is not observed in Figures 9-12. In this case, depending on the values of a and b, the value of $t -$ (time for the solution to reach a fixed point) changes.

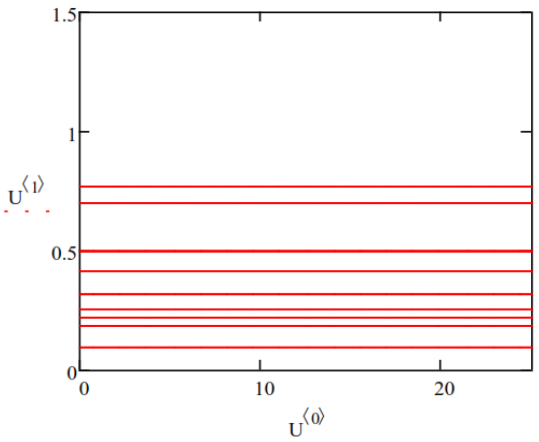

Figure 7

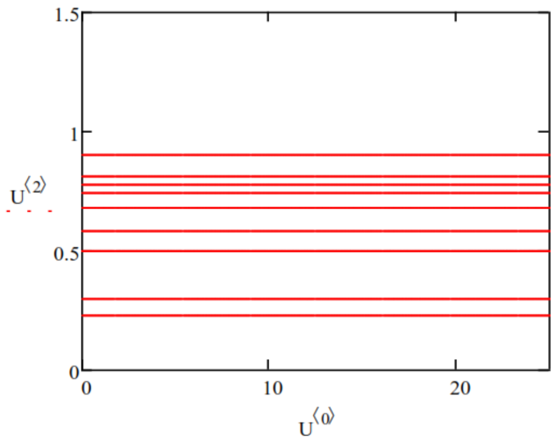

Figure 8

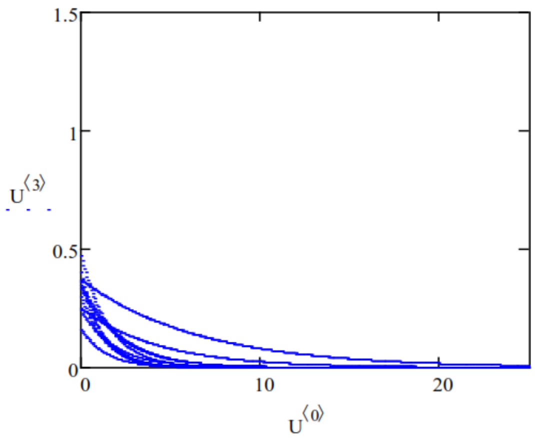

Figure 9

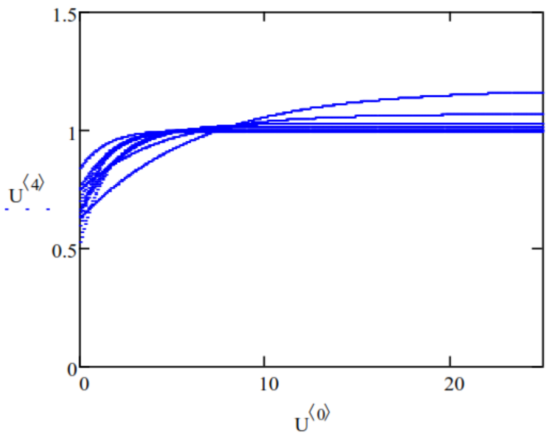

Figure 10

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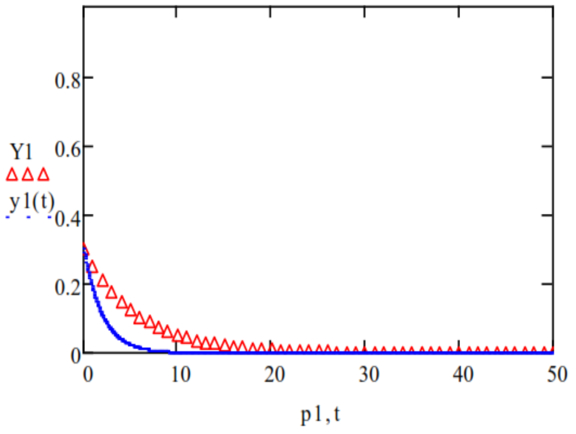

Figure 11

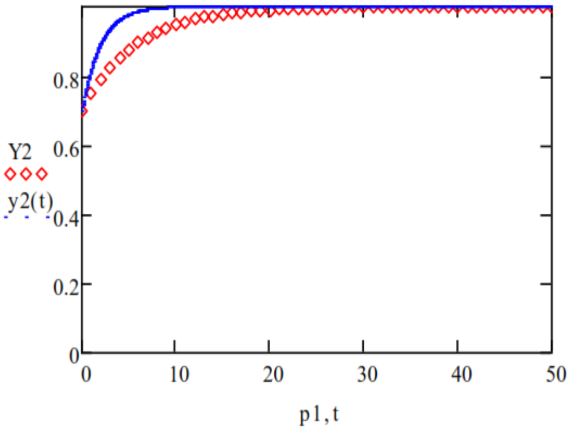

Figure 12

(2) we compare analytic and numerical solutions of the system (4) condition (Graphs 11-12). Since the solutions $x_1(t)$ and $x_2(t)$ are completely superimposed, their graphs were not quoted.

The graphs are drawn with the following initial conditions $C3 = (0.75, 0.25, 0.3, 0.7)$ and $a = 1.1,\ b = 0.7$.

Thus, the following theorem is proved.

Theorem. 1^0. The general solution of system (2) is in the form (3) and $M_1 -$ is unstable, $M_1 -$ is a stable (but not asymptotically stable) fixed point.

2^0. If $y_1(t) + y_2(t) = 1$ is taken into account and condition (4) is fulfilled, then

$$\lim_{t\to+\infty} Y(t) = \lim_{t\to+\infty} \big(y_1(t), y_2(t)\big) = (0,1).$$

4. Conclusion

From the above, we can conclude that scientists often believe that in the biological models they study, it is mainly the «pull» equilibrium points that are important, since they can be observed by the real system under the influence of changes. This is also evident in these questions studied.

Graph 10 also shows that the value of $y_2(t)$ is greater than 1. If the initial conditions and a and b are chosen in such a way that they satisfy relation (4), the condition of Theorem 2^0 is also satisfied.

Declarations

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This study has not received any funds from any organization.

Conflict of Interest

The author declares that she has no conflict of interest.

Consent for Publication

The author declares that she consented to the publication of this study.

Authors' Contribution

Author's individual contribution.

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References

[1] Petrovsky, I.G. (2020). Lectures on the theory of ordinary differential equations [in Russian]. Moscow, LitRes.

[2] Ethan Akin & Viktor Losert (1984). Evolutionary dynamics of zero-sum games. J. Math. Biology, 20: 231–258.

[3] Xia Liu & Yepeng Xing (2013). Bifurcations of a Ratio-Dependent Holling-Tanner System with Refuge and Constant Harvesting. Abstract and Applied Analysis, Pages 1–10. doi: http://dx.doi.org/10.1155/2013/478315.

[4] Bratus, A.S., Novozhilov, A.S., & Platonov, A.P. (2009). Dynamic systems and models of biology. Moscow, FIZMATLIT, [in Russian].

[5] Juan Liu, Zizhen Zhang & Ming Fu (2012). Stability and Bifurcation in a Delayed Holling-Tanner Predator-Prey System with Ratio-Dependent Functional Response. Journal of Applied Mathematics, Pages 1–19. doi: 10.1155/2012/384293. 2012.

[6] Zhiqing Liang & Hongwei Pan (2007). Qualitative analysis of a ratio dependent Holling-Tanner model. J. Math. Anal. Appl., 334: 954–964.

[7] Rasulov, X.R. (2022). Qualitative analysis of strictly non-Volterra quadratic dynamical systems with continuous time. Communications in Mathematics, 30(1): 239–250.

[8] Rasulov, Kh.R. (2018). On a continuous time F-quadratic dynamical system. Uzbek Math. Journal, 4: 126–130.

[9] Pascal, R.D. (2013). Dynamical Systems and Models of Continuous Time. The Oxford Handbook of Quantitative Methods in Psychology, Volume 2.

[10] Tahir Ahmad & Tan Lit Ken (2011). Flows of Continuous-Time Dynamical Systems with No Periodic Orbit as an Equivalence Class under Topological Conjugacy Relation. Journal of Mathematics and Statistics, 7(3): 207–215.

[11] Isaev, A.S., Hlebopros, R.G., Nedorezev, L.V., et al. (2001). Population dynamics of forest insects [in Russian]. Nauka, Moscow.

[12] Rasulov, X.R., Yashieva & F. Yu. (2021). On some Volterra quadratic stochastic operators of a two-sex population with continuous time. Science, Technology and Education [in Russian], 72(2-2): 23–26.

[13] Nedorezov, L.V. (1986). Modeling of mass reproductions of forest insects. Nauka, Novosibirsk [in Russian].

[14] Rozikov, U.A., & Zhamilov, U.U. (2008). F-quadratic stochastic operators. Math. Notes, 83(4): 554–559.

[15] Lyubich Yu, I. (1992). Mathematical Structures in Population Genetics (Mathematics, 22). Springer-Verlag, Berlin.

[16] Rozikov, U.A., & Djamilov, U.U. (2011). Volterra quadratic stochastic operators of a two-sex population. Ukrainian Mathematical Journal [in Russian], 63(17): 985–998.